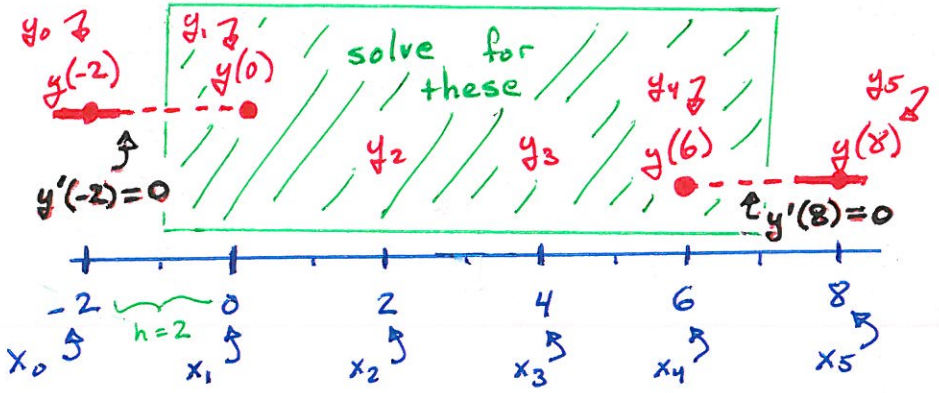
Discretize $-y'' = x^2$ with $\begin{cases} y'(-2) = 0 \\ y'(8) = 0 \end{cases}$ $h = 2$.

Note: $h = 2$. This problem is silly.

Note: Boundary values involve derivatives! (This DE might not have a solution...)



Boundary conditions:

- $y'(-2) = 0$ ⇝ {first two y-values are same} ⇝ $\boxed{y_0 = y_1}$
- $y'(8) = 0$ ⇝ {last two y-values are same} ⇝ $\boxed{y_4 = y_5}$

Write equations at x_1, x_2, x_3, x_4

$$\begin{array}{rcll} & -y'' & = & x^2 \\ -\tfrac{1}{2^2}(\cancel{y_0}^{\,y_1} - 2y_1 + y_2) & = & 0^2 & (x_1 = 0) \\ -\tfrac{1}{2^2}(y_1 - 2y_2 + y_3) & = & 2^2 & (x_2 = 2) \\ -\tfrac{1}{2^2}(y_2 - 2y_3 + y_4) & = & 4^2 & (x_3 = 4) \\ -\tfrac{1}{2^2}(y_3 - 2y_4 + \cancel{y_5}^{\,y_4}) & = & 6^2 & (x_4 = 6) \end{array}$$

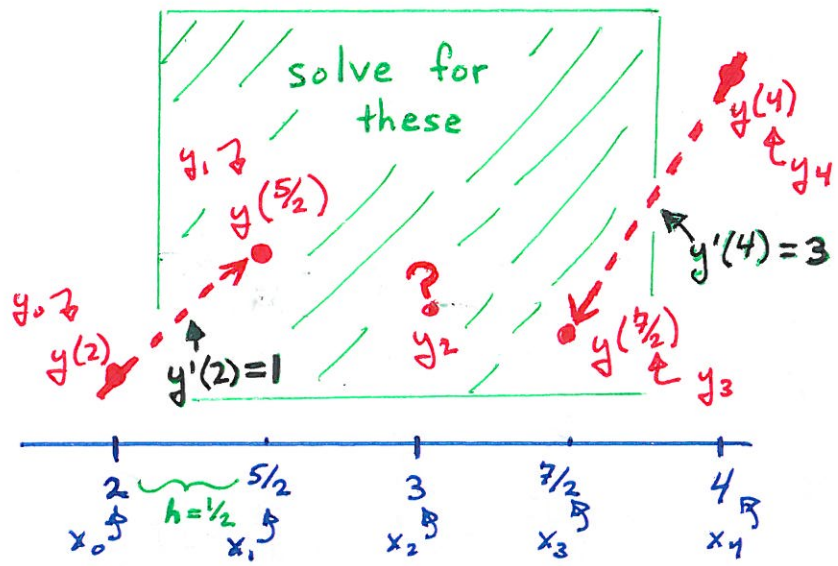
Simplify:

$$\begin{array}{rcl} \tfrac{1}{4}y_1 - \tfrac{1}{4}y_2 & = & 0 \\ -\tfrac{1}{4}y_1 + \tfrac{1}{2}y_2 - \tfrac{1}{4}y_3 & = & 4 \\ -\tfrac{1}{4}y_2 + \tfrac{1}{2}y_3 - \tfrac{1}{4}y_4 & = & 16 \\ -\tfrac{1}{4}y_3 + \tfrac{1}{4}y_4 & = & 36 \end{array}$$

Convert to matrix:

$$\begin{bmatrix} \tfrac{1}{4} & -\tfrac{1}{4} & 0 & 0 \\ -\tfrac{1}{4} & \tfrac{1}{2} & -\tfrac{1}{4} & 0 \\ 0 & -\tfrac{1}{4} & \tfrac{1}{2} & -\tfrac{1}{4} \\ 0 & 0 & -\tfrac{1}{4} & \tfrac{1}{4} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 16 \\ 36 \end{bmatrix}$$

Discretize $2y'' = 4x + 1$ w/ $\begin{cases} y'(2)=1 \\ y'(4)=3 \end{cases}$ $h = 1/2$



Boundary conditions:

- $y'(2)=1$ ⇝ {slope = 1 between first two y} ⇝ $\frac{y_1 - y_0}{1/2} = 1$

$$y_0 = y_1 - 1/2$$

- $y'(4)=3$ ⇝ {slope = 3 between last two y} ⇝ $\frac{y_4 - y_3}{1/2} = 3$

$$y_4 = y_3 + 3/2$$

Write equations at x_1, x_2, x_3:

$$2y'' = 4x + 1$$

$$\begin{aligned} 2\,1/(1/2)^2 (y_0 - 2y_1 + y_2) &= 4 \cdot 5/2 + 1 \quad (x_1 = 5/2) \\ 2\,1/(1/2)^2 (y_1 - 2y_2 + y_3) &= 4 \cdot 3 + 1 \quad (x_2 = 3) \\ 2\,1/(1/2)^2 (y_2 - 2y_3 + y_4) &= 4 \cdot 7/2 + 1 \quad (x_3 = 7/2) \end{aligned}$$

Boundary conditions ⇒ $\begin{cases} y_0 = y_1 - 1/2 \\ y_4 = y_3 + 3/2 \end{cases}$

$$\begin{aligned} 8((y_1 - 1/2) - 2y_1 + y_2) &= 11 \\ 8(y_1 - 2y_2 + y_3) &= 13 \\ 8(y_2 - 2y_3 + (y_3 + 3/2)) &= 15 \end{aligned}$$

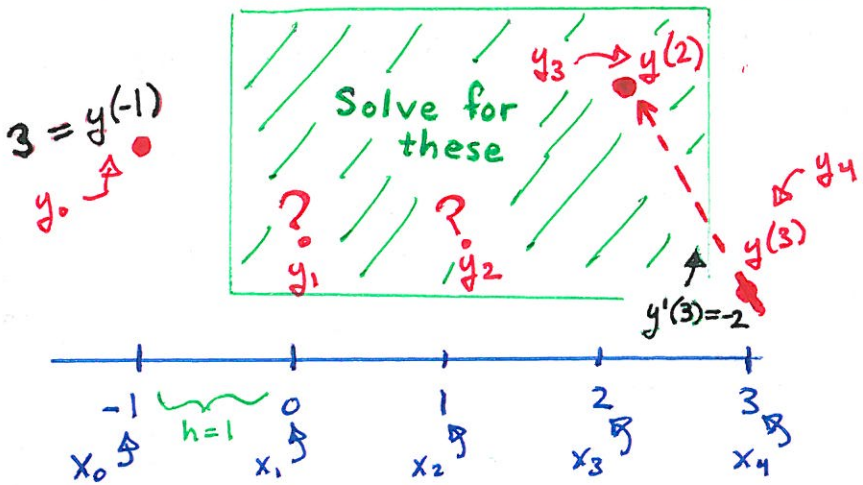
$$\begin{aligned} -8y_1 + 8y_2 &= 11 + 4 \\ 8y_1 - 16y_2 + 8y_3 &= 13 \\ 8y_2 - 8y_3 &= 15 - 12 \end{aligned}$$

Matrix:

$$\begin{bmatrix} -8 & 8 & 0 \\ 8 & -16 & 8 \\ 0 & 8 & -8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 13 \\ 3 \end{bmatrix}$$

Discretize $(x+1)y'' + y = \delta(x-1)$ $\begin{cases} y(-1)=3 \\ y'(3)=-2 \end{cases}$ $h=1$

Note: One boundary condition has a derivative, other condition doesn't



Boundary conditions

- $y(-1) = 3 \rightsquigarrow y_0 = 3$
- $y'(3) = -2 \rightsquigarrow$ $\begin{cases} \text{slope} = -2 \\ \text{between} \\ \text{last two } y \end{cases}$

$$\rightsquigarrow \frac{y_4 - y_3}{1} = -2$$

$$\rightsquigarrow \boxed{y_4 = y_3 - 2}$$

Write equations at x_1, x_2, x_3:

$$(x+1)\cdot y'' + y = \delta(x-1)$$

$$\begin{bmatrix} 1\cdot \frac{1}{(1)^2}(y_0 - 2y_1 + y_2) + y_1 = \delta(0-1) \to 0 & (x_1=0) \\ 2\cdot \frac{1}{(1)^2}(y_1 - 2y_2 + y_3) + y_2 = \delta(1-1) \to 1/1 & (x_2=1) \\ 3\cdot \frac{1}{(1)^2}(y_2 - 2y_3 + y_4) + y_3 = \delta(2-1) \to 0 & (x_3=2) \end{bmatrix}$$

Boundary conditions $\Longrightarrow$ $\begin{cases} y_0 = 3 \\ y_4 = y_3 - 2 \end{cases}$

$$\begin{bmatrix} (3 - 2y_1 + y_2) + y_1 = 0 \\ 2(y_1 - 2y_2 + y_3) + y_2 = 1 \\ 3(y_2 - 2y_3 + (y_3 - 2)) + y_3 = 0 \end{bmatrix}$$

$$\begin{bmatrix} -y_1 + y_2 = 0 - 3 \\ 2y_1 - 3y_2 + 2y_3 = 1 \\ 3y_2 - 2y_3 = 0 + 6 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 & 0 \\ 2 & -3 & 2 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 6 \end{bmatrix}$$